WESLEY COLLEGE
By daring & by doing

YEAR 12 MATHEMATICS SPECIALIST
SEMESTER ONE 2017
QUESTIONS OF REVIEW 1: Polynomials & Polars

Name: Answers

Friday 17h February | Time: 30 minutes | Mark /30

ave 27.3

Calculator free.

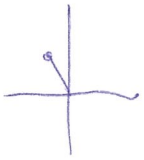
1. [4 marks – 2 each]

Convert:

a) $-2-2i$ to polars (r,θ^r) $\left(2\sqrt{2}, -\frac{3\pi}{4}\right)$

b) $\frac{1}{2}\text{cis}\left(\frac{2\pi}{3}\right)$ to rectangular co-ordinates

$\left(-\frac{1}{4}, \frac{\sqrt{3}}{4}\right)$



2. [4 marks – 1 each]

For $z=3-4i$, evaluate:

a) $|z|$ 5

b) z^2 $(3-4i)(3-4i) = 9-24i-16 = -7-24i$

c) $z\times\bar{z}$ $(3-4i)(3+4i) = 9+16 = 25$

d) $\frac{\bar{z}}{z}$, with a real denominator

$\frac{3+4i}{3-4i} = \frac{-7+24i}{25}$ $\left(\times \frac{3+4i}{3+4i}\right)$

3. [6 marks – 1, 2, 1, 1, 1]

If $z = 4\text{cis}\left(-\frac{\pi}{3}\right)$ and $\omega = 2\text{cis}\left(\frac{5\pi}{6}\right)$, determine, in *cis* form:

a) ωz $= 8 \text{ cis } \frac{\pi}{2}$

b) $\frac{z}{\omega}$ $= 2 \text{ cis}\left(-\frac{7\pi}{6}\right) = 2 \text{ cis}\left(\frac{5\pi}{6}\right)$

c) $\frac{z}{i}$ $= 4 \text{ cis}\left(\frac{5\pi}{6}\right)$

d) $\overline{z}$ $= 4 \text{ cis}\left(\frac{\pi}{3}\right)$

e) $\frac{12}{z}$ $= 3 \text{ cis}\left(\frac{\pi}{3}\right)$

4. [8 marks – 1, 1, 1, 1, 1, 2, 1]

Two complex numbers, z and ω are shown on the Argand diagram. Add each of these to this diagram:

a) $2z$

b) z^2

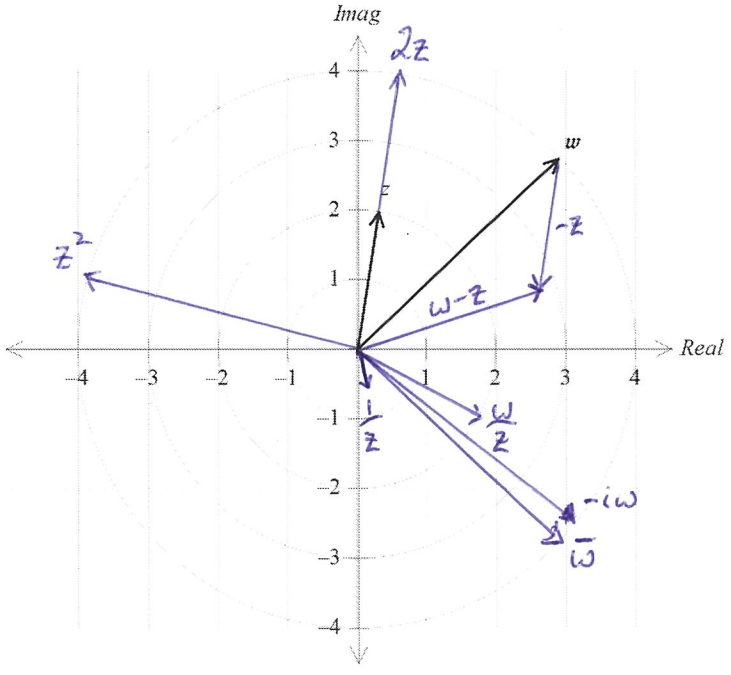
c) $\omega - z$

d) $\frac{\omega}{z}$

e) $\overline{\omega}$

f) $\frac{1}{z}$

g) $-i\omega$



5. [8 marks – 1, 1, 1, 2, 3]

For $P(z) = z^3 - 6z^2 + az - 10$

a) express $P(2)$ in terms of a

$P(2) = 8 - 24 + 2a - 10 = 2a - 26$

b) determine the remainder, in terms of a, when $P(z)$ is divided by $(z-2)$

$2a - 26$

c) evaluate a if $P(2) = 0$:

$a = 13$

d) write a polynomial expression for $\frac{P(z)}{z-2}$

$= z^2 - 4z + 5$

	1	−6	13	−10
$z=2$	1	−4	5	0

e) find all the roots of $P(z)$

$(z-2)(z^2-4z+5) = 0$

$\Rightarrow z = 2$ or $\frac{4 \pm \sqrt{16-20}}{2}$

$\Rightarrow z = 2$ or $2 \pm i$